Why there is Something rather than Nothing (from Everything):

origin of the cosmological constant and dark energy

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Introduction

EQG

- cosmological wavefunctions
- baby universes
- BH thermodynamics

Problems:

- indefiniteness of the Euclidean gravitational action;
- infrared catastrophe of small cosmological constant $\Lambda$.

$$\Psi_{HH} \sim \exp \left( \frac{3\pi}{2G\Lambda} \right) \to \infty, \quad \Lambda \to 0;$$

- cosmology debate of the no-boundary vs tunneling proposals

EQG density matrix of the Universe

- elimination of the infrared catastrophe of small $\Lambda$
- ensemble of universes in a limited range of $\Lambda$
- selection mechanism for string landscape


Justification of these results from Lorentzian quantum gravity (LQG) and suggestion of a new mechanism of dark energy (A.B., hep-th/0704.0083)
Plan

- EQG density matrix:
  effects of conformal anomaly and radiation — limiting the cosmological constant ensemble

- LQG density matrix — microcanonical ensemble

- Sum over Everything

- Conformal rotation in EQG — selection of thermal instantons

- Dark energy from the microcanonical state of the Universe:
  hierarchy problem, strings and evolving extra dimensions

- Conclusions: Something rather than Nothing comes from Everything
From the pure Hartle-Hawking state to a statistical ensemble – the density matrix:

\[ |\Psi_{HH}\rangle = \Psi_{HH}[\varphi] \]

\[ |\Psi_{HH}\rangle\langle\Psi_{HH}| = \rho_{HH}[\varphi, \varphi'] \]

\[ \hat{\rho}_{\text{mixed}} = \rho_{\text{mixed}}[\varphi, \varphi'] \]

instanton bridge mediates density matrix correlations
EQG density matrix

\[ \rho[\varphi, \varphi'] = e^{\Gamma} \int_{g, \phi \mid \Sigma, \Sigma'} D[g, \phi] \exp \left( -S_E[g, \phi] \right) \mid_{\Sigma, \Sigma'}^{\Sigma, \Sigma'} = (\varphi, \varphi') \]

\( \text{tr} \rho = 1 \)

Effective action: statistical sum

\[ e^{-\Gamma} = \int_{g, \phi \mid \Sigma} D[g, \phi] \exp \left( -S_E[g, \phi] \right) \mid_{\Sigma}^{\Sigma'} \]

integration over periodic fields on a torus:

D. Page (1998)
Euclidean FRW metric

\[ ds^2 = N^2 d\tau^2 + a^2 d^2\Omega^{(3)}, \]

\[ \text{3-sphere of a unit size} \]

\[ \begin{bmatrix} g, \phi \end{bmatrix} = \begin{bmatrix} a(\tau), N(\tau); \Phi(x) \end{bmatrix} \]

\[ \Phi(x) = (\varphi(x), \psi(x), A_\mu(x), h_{\mu\nu}(x), \ldots) \]

quantum "matter" – cosmological perturbations

\[ e^{-\Gamma} = \int \text{D}[a, N] \ e^{-\Gamma_E[a, N]} \]

periodic

\[ e^{-\Gamma_E[a, N]} = \int \text{D}\Phi(x) \ e^{-S_E[a, N; \Phi(x)]} \]

quantum effective action of \( \Phi \) on minisuperspace background
Effective action for conformally coupled fields

\[ \Gamma_E[a, N] = \int d\tau N \mathcal{L}(a, a') + F(\eta) \]

\[ \mathcal{L}(a, a') = -a a'^2 - a + H^2 a^3 + B \left( \frac{a'^2}{a} - \frac{a'^4}{6a} + \frac{1}{2a} \right) \]

local part \quad nonlocal (thermal) part

\[ a' = \frac{1}{N} \frac{da}{d\tau}, \quad \Lambda = 3H^2 \]

Hubble constant

Classical part \quad conformal anomaly part \quad vacuum (Casimir) energy

\[ B \quad \text{coefficient of the Gauss-Bonnet term in the total conformal anomaly} \]

Nonlocal part -- free energy

\[ F(\eta) = \pm \sum \omega \ln \left( 1 \mp e^{-\omega \eta} \right), \quad \eta = \int d\tau \frac{N}{a} \quad \text{instanton period in units of conformal time --- inverse temperature} \]

energies of field oscillators on a 3-sphere
Saddle points of the path integral — solutions of Euclidean effective equation of motion

\[ \frac{\delta \Gamma_E[a, N]}{\delta N(\tau)} = 0 \]

quantum anomaly term

\[ \frac{a^2}{a^2} + B \left( \frac{1}{2} \frac{a^4}{a^4} - \frac{a^2}{a^4} \right) = \frac{1}{a^2} - H^2 - \frac{C}{a^4} \]

\[ C = \frac{B}{2} + \frac{dF(\eta)}{d\eta}, \quad \eta = \int d\tau \frac{N}{a} \]

amount of radiation constant

Flirouzjahi, Sarangi & Tye (2004); Sarangi & Tye (2005); Brustein & de Alwis (2006)

Solutions — set of tubular periodic garland-type instantons with oscillating scale factor

1-fold, \( k=1 \)

k-folded garland, \( k=1, 2, 3, \ldots \)

Halliwell & Myers (1999); Fischler, Morgan & Polchinski (1990)
\[ 4C H^2 < 1 \]
\[ C \geq B - B^2 H^2 \]
\[ BH^2 \leq \frac{1}{2} \]

- \( \Lambda_{\text{min}} < \Lambda < \Lambda_{\text{max}} \) bounded range of the cosmological constant \( \longrightarrow \) selection rule for string landscape
- for any \( \Lambda > 0 \) elimination of the infrared catastrophe (anomaly effect)
- \( \Lambda_{\text{max}} = 3m_P^2/2B \) new quantum gravity scale (anomaly effect)
- \( \Lambda_{\text{min}}, \Lambda_{\text{max}} \rightarrow \frac{\Lambda_{\text{min}}}{N}, \frac{\Lambda_{\text{max}}}{N} \), \( N = \# \) of quantum fields: \( 1/N \)-approximation
LQG density matrix

Representation of three-metric and matter fields \(q = (g_{ij}(x), \phi(x))\); \(p\) — conjugated momenta

\[
\rho(q_+, q_-) = e^{\Gamma} \int D[q, p, N] \exp\left(i \int_{t_-}^{t_+} dt \left( p \dot{q} - N^\mu H_\mu \right) \right)
\]

Range of integration over \(N^\mu\): \(-\infty < N^\mu < \infty\)

\[
\hat{H}_\mu(q, \partial/\partial q) \rho(q, q_-) = 0
\]

Microcanonical density matrix

\[
\hat{\rho} \sim "\left( \prod_\mu \delta(\hat{H}_\mu) \right)"
\]
A simplest analogy — an unconstrained system with a conserved Hamiltonian $\hat{H}$ in the microcanonical state with a fixed energy $E$, 

$$\tilde{\rho} \sim \delta(\hat{H} - E)$$

Spatially closed cosmology does not have freely specifiable constants of motion. The only conserved quantities are the Hamiltonian and momentum constraints $H_\mu$, all having a particular value — zero.

The microcanonical ensemble with 

$$\tilde{\rho} \sim \left( \prod_\mu \delta(\bar{H}_\mu) \right)$$

is as a most general and natural candidate for the quantum state of the closed Universe.

The microcanonical statistical sum of the Universe is just a uniformly distributed (with a unit weight) integral over entire phase space of true physical degrees of freedom — Sum over Everything.
Sum over Everything

Statistical sum
\[ e^{-\Gamma} = e^{-\Gamma} \operatorname{Tr}_{\text{phys}} \bar{\rho} = \int \ D[q,p,N] \ e^{i \int dt (p \dot{q} - N^\mu H_\mu)} \]
periodic

Physical reduction in the unitary gauge, \( \chi^\mu(q,p) = 0 \), \( (q,p) \rightarrow (q_{\text{phys}}, p_{\text{phys}}) \).

Canonical Faddeev-Popov integral in terms of physical variables:
\[
\int D[q,p,N] \ e^{i \int dt (p \dot{q} - N^\mu H_\mu)} = \int Dq_{\text{phys}} DP_{\text{phys}} \ e^{i \int dt (p_{\text{phys}} \dot{q}_{\text{phys}} - H_{\text{phys}}(t))} = \operatorname{Tr}_{\text{phys}} \left( T e^{-i \int dt \hat{H}_{\text{phys}}(t)} \right)
\]
chronological ordering

On-shell Faddeev-Popov path integral is gauge-independent.

In static gauges, \( \partial_t \chi^\mu(q,p,t) = 0 \), \( \hat{H}_{\text{phys}}(t) = 0 \) (closed cosmology)

\[ e^{-\Gamma} = \operatorname{Tr}_{\text{phys}} I_{\text{phys}} = \int dq_{\text{phys}} dp_{\text{phys}} = \text{sum over Everything.} \]
Gaussian integration over momenta —
Lagrangian path integral:
\[ e^{-\Gamma} = \int D[q, N] \ e^{iS_L[q, N]} \]

\[ e^{-\Gamma} = \int D[a_L, N_L] \ e^{i\Gamma_L[a_L, N_L]} \]
\[ e^{i\Gamma_L[a_L, N_L]} = \int D\Phi_L(x) \ e^{iS_L[a_L, N_L; \Phi_L(x)]} \]

Field decomposition: \([q, N] \rightarrow [a_L(t), N_L(t); \Phi_L(x)], \quad D[q, N] = D[a_L, N_L] \times D\Phi_L(x)\)

LQG path integrals with real integration variables

No periodic solutions of Lorentzian effective equation in real time and real geometry!
Saddle points of the Lorentzian path integral exist in Wick-rotated (Euclidean) geometry:

\[ t = \tau, \quad N_L = -iN, \quad iS_L[a_L, N_L; \Phi_L] = -S_E[a, N; \Phi], \]

\[ e^{i\Gamma_L[a_L, N_L]} = e^{-\Gamma_E[a, N]} \]
\[ e^{i\Gamma} = e^{-\Gamma_E} \]

EQG path integral with the imaginary lapse integration contour:
\[ N \in [-i\infty, i\infty] \]
Steepest decent integration contour for the Euclidean lapse — conformal rotation in EoG

\[ N = iN_L \]

Deformation of the original contour of integration

\[ -\infty < N_L < \infty \]

into the complex plane to pass through the saddle point

\[ N = 1 \text{ (Euclidean saddle point)} \]
Conformal rotation in the one-loop approximation — selection of thermal instantons with a fixed Euclidean time period.

Integration measure in the Faddeev-Popov path integral:

\[ D[a, N] = D a D N \mu[a, N] \delta[\chi] \text{Det } Q \]

Local Lagrangian in the action:

\[ \mu_{\text{1-loop}} = \prod_{\tau} \left( \frac{\partial^2 (N \mathcal{L})}{\partial a \partial \dot{a}} \right)^{1/2} = \prod_{\tau} \left( \frac{D}{N a^2 a'^2} \right)^{1/2} \]

\[ D = a a'^2 (a^2 - B + B a'^2) > 0 \]

on background instantons

The gauge disentangling conformal mode perturbations:

\[ \chi(a, N) = \delta N - (N/a) \delta a \]

The Faddeev-Popov operator:

\[ Q = a (d/d\tau)a^{-1} \]

Perturbations on background

Quadratic part of the action in terms of the conformal mode \( \sigma \):

\[ \delta^2 \Gamma_E = -\frac{3\pi m_P^2}{2} \int d\tau ND \left[ \left( \frac{\sigma}{a'} \right)^2 \right] < 0 \]

But the integration range over \( \sigma \) is imaginary!
Density Matrix Reloaded:

Minimum set of assumptions — an ultimate equipartition in physical phase space in the form of the microcanonical state of closed quantum cosmology

- Constraining the ensemble of $\Lambda$ (and possibly landscape of string vacua)
- A new dark energy mechanism transcending the inflationary and matter-domination stages as a quasi-equilibrium decay of the initial microcanonical state

``Nothing comes from Nothing``

Sidney Coleman: ``There is Nothing rather than Something``

Something (rather than Nothing) comes from Everything
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Mechanism of variable $B$ from string/Kaluza-Klein theory

$B \Rightarrow B(t)$ indefinitely growing with evolving size of X-tra dimension(s) $R(t)$

$B$ — coefficient of the Gauss-Bonnet term in the total conformal anomaly;

$B \sim \mathcal{N}$ number of conformally invariant massless modes — KK and winding modes

Masses of KK and winding modes:

$$m_{n,w}^2 = \frac{n^2}{R^2} + \frac{w^2}{\alpha'^2 R^2} \ll H_+^2 \sim \frac{m_p^2}{\mathcal{N}}$$

- Growing tower of superhorizon KK modes ($w = 0, \ n \leq \mathcal{N}$):

  $$\mathcal{N} \sim (m_p R)^{2/3} \Rightarrow H_+ \sim \frac{m_p}{(m_p R)^{1/3}} \text{ indefinitely decreases with } R \to \infty$$

- Growing tower of superhorizon winding modes ($n = 0, \ w \leq \mathcal{N}$):

  $$\mathcal{N} \sim \left(\frac{m_p \alpha'}{R}\right)^{2/3} \Rightarrow H_+ \sim m_p \left(\frac{R}{m_p \alpha'}\right)^{1/3} \text{ indefinitely decreases with } R \to 0$$

$$\Lambda \sim m_p^2 \Rightarrow \Lambda_{\text{present}} \quad \text{(fine-tuning the expansion/contraction of X-tra space size?!) }$$
Mechanism of variable $B$ from string/Kaluza-Klein theory

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Masses of KK and winding modes:

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• Growing tower of superhorizon KK modes $(w = 0, \, n < \mathcal{N})$.

$\mathcal{N} \sim (m_p R)^{2/3} \Rightarrow H_+ \sim \frac{m_p}{(m_p R)^{1/3}}$ indefinitely with $R \to \infty$

• Growing tower of superhorizon winding modes $(n = \mathcal{N})$

$\mathcal{N} \sim \left(\frac{m_p \alpha'}{R}\right)^{2/3} \Rightarrow H_+ \sim m_p \left(\frac{R}{m_p \alpha'}\right)^{1/3}$ indefinitely with $R \to 0$

$\Lambda \sim m_p^2 \Rightarrow \Lambda_{\text{present}}$ (fine-tuning the expansion/contraction of X-tra space size?!)
Mechanism of variable $B$ from string/Kaluza-Klein theory

$B \Rightarrow B(t)$ \quad indefinitely growing with evading size of $X$ (by dimensional $R(t)$

$B$ \quad coefficient of the Gauss-Bonnet term in the total conformal anomaly

$B \sim \mathcal{N}$ \quad number of conformally invariant \textit{superdense} modes \quad $-\mathcal{N}$ and $\mathcal{N}$ and winding modes

Masses of $KK$ and winding modes:

$$m^2 = \frac{n^2}{R^2} + \frac{w^2}{\alpha^2} R^2 < H^2 \sim \frac{m_p^2}{\mathcal{N}}$$

- \text{Growing tower of superdense KK modes} \quad (w = 0, n \leq \mathcal{N})
  \quad $N \sim (m_p R)^{2/3} \Rightarrow H_k \sim \left(\frac{m_p}{(m_p R)^{1/3}}\right)$ \quad indefinitely decreases with $R \rightarrow 0$
- \text{Growing tower of superdense winding modes} \quad (n = 0, w \leq \mathcal{N})
  \quad $N' \sim \left(\frac{m_p R}{\alpha^2}\right)^{2/3} \Rightarrow H_k \sim \left(\frac{R}{m_p \alpha}\right)^{1/3}$ \quad indefinitely decreases with $R \rightarrow 0$

$\Lambda \sim m_p^2 \Rightarrow \Lambda_{\text{present}}$ \quad (the tuning the expansion contraction of $X$ space zero?)
Mechanism of variable $B$ from string/Kaluza-Klein theory

$B \Rightarrow B(t)$ indefinitely growing with evading size of $X$ (variable constant $R(t)$)

$B$ coefficient of the Gauss-Bonnet term in the total conformal anomaly,

$B \sim N$ number of conformal invariant *modular* modes — *K* and winding modes

Masses of $K$ and winding modes:

$$m^2_{K,\omega} = \frac{n^2}{R^2} + \frac{w^2}{\alpha^2} R^2 < H^2 \sim \frac{m^2}{N}$$

- Growing tower of *supersymmetric* $K$-modes ($w = 0$, $n \leq N$):
  $$N \sim (m_p R)^{2/3} \Rightarrow H_k \sim \frac{m_p}{(m_p R)^{1/3}}$$  \quad (indefinitely decreases with $R \to \infty$)

- Growing tower of *supersymmetric* winding modes ($n = 0$, $w \leq N$):
  $$N' \sim (m_p n^2 R)^{2/3} \Rightarrow H_k \sim m_p (\frac{R}{m_p \alpha})^{1/3}$$  \quad (indefinitely decreases with $R \to 0$)

$\Lambda \sim m^2_p \Rightarrow \Lambda_{\text{present}}$ (the tuning the expansion/contraction of $X$ [space $X^2$]?)
$$e^{-\Gamma_{\text{loop}}} = e^{-\Gamma_0} \text{Det} Q_0 \int_{\mathcal{D}} D\sigma \left( \prod_{i} \frac{1}{\sqrt{D_{\sigma}}} \right)^{1/2} e^{-\frac{1}{2} \bar{Q}_0 \bar{Q}}$$

ghost factor

confinement mode contribution factor

$$= e^{-\Gamma_0} \text{Det} \left( \frac{d}{dr} \right) \left[ \text{Det} \left( \frac{1}{\sqrt{D}} \frac{d}{dr} D \frac{d}{dr} \sqrt{D} \right) \right]^{-1/2} = e^{-\Gamma_0}$$

private boundary condition

II

1

no quantum corrections from integration

selection of saddle point instantons
\begin{align*}
e^{-\Gamma_{1\text{-loop}}} &= e^{-\Gamma_0} \text{Det} \mathcal{Q}_0 \int_{-\infty}^{\infty} D\sigma \left( \prod_{\tau} D/a'2 \right)^{1/2} e^{-\frac{1}{2} a'2 \Gamma_E} \\
 &= e^{-\Gamma_0} \times \text{Det} \left( \frac{d}{d\tau} \right) \left[ \text{Det} \left( -\frac{1}{\sqrt{D}} \frac{d}{d\tau} \frac{d}{d\tau} \frac{d}{d\tau} \frac{d}{d\tau} \frac{d}{d\tau} \frac{d}{d\tau} \frac{d}{d\tau} \frac{d}{d\tau} \right) \right]^{-1/2} = e^{-\Gamma_0} \\
 &\hspace{2cm} \text{ghost factor} \hspace{2cm} \text{conformal mode contribution factor} \\
 &\hspace{2cm} \text{periodic boundary conditions} \hspace{2cm} \text{no quantum corrections from } \alpha\text{-integration} \hspace{2cm} \text{--- selection of saddle-point instantons}
\end{align*}
Dark energy from the microcanonical state of the Universe

Lorentzian Universe with initial conditions set by the saddle-point instanton

Analytic continuation of the instanton solutions:
\[ \tau = it, \quad a_L(t) = a(it) \]

Two quasi-exponential branches of the evolution (analogue of DGP model):
\[ a_L(t) \sim e^{H_{\pm}t}, \quad t \to \infty, \quad H_{\pm}^2 = \frac{m_P^2}{B} \left( 1 \pm \sqrt{1 - 2BH^2/m_P^2} \right) \]

Decay of a composite H in the end of inflation:
\[ H^2 \to 0, \quad H_+^2 \to \frac{2m_P^2}{B} \]

Cosmological acceleration with
\[ \Lambda_+ = 4\Lambda_{\text{max}} = \frac{6m_P^2}{B} \quad \text{--- new QG scale (upper bound on } \Lambda) \]
Hierarchy problem, strings and extra dimensions

• Early Universe: constraints from large-scale structure formation

\[ \Lambda_{\text{GUT}} \simeq \Lambda_{\text{early}} \ll \Lambda_{\text{Planck}} \]
\[ (10^{16}\ \text{GeV})^2 \quad (10^{19}\ \text{GeV})^2 \]

\[ \Lambda_{\text{present}} \simeq 0.7 \varepsilon_{\text{crit}} \]
\[ (10^{-33}\ \text{eV})^2 \]

\[ B_{\text{early}} \quad B_{\text{present}} \quad B = B(t) \]

• Present Universe: cosmological acceleration or dark energy

should be a function of time

String theory vs EQG density matrix:

Limited instanton ensemble is generated due to the nonlocal infrared effect of the conformal anomaly
— should fit into string theory at its low energy field-theoretic level

Constraining the landscape of string vacua:

\[ \# \text{ of string vacua} \quad 10^{250} \div 10^{500} \quad \text{vs} \quad \Lambda_{\text{min}} < \Lambda < \Lambda_{\text{max}} \]
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$m_{n,w}^2 = \frac{n^2}{R^2} + \frac{w^2}{\alpha'^2} R^2 \ll H_+^2 \sim \frac{m_P^2}{\mathcal{N}}$

$\Rightarrow$ approximate masslessness and conformal invariance

- Growing tower of superhorizon KK modes $(w = 0, \ n \leq \mathcal{N})$:

$\mathcal{N} \sim (m_P R)^{2/3} \Rightarrow H_+ \sim \frac{m_P}{(m_P R)^{1/3}}$ indefinitely decreases with $R \to \infty$

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